Order in the particle zoo V4

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Abstract

The standard model of physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived from the relationship of a point charge and a photon representation of energy. Necessary input parameters are square of the elementary charge divided by electric constant and one model specific constant. The value of α itself can be approximated numerically by the gamma functions of the integrals involved.

The model provides a simple qualitative explanation why leptons, most notably the tauon, are not subject to strong interaction.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles" [1]. The standard model of physics [2] divides these particles into leptons, considered to be the fundamental "elementary particles" and the hadrons, composed of two (mesons) or three (baryons) quarks. Well hidden in the data of particle energies lies another ordering principle which can be derived by interpreting particles as electromagnetic objects. There exist numerous attempts to calculate particle energy with electromagnetic models going back as far as 1881 with the work of J.J.Thomson [3] [4]. In the work presented here, to obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, $\Psi(r, e^2/\varepsilon, \rho, \alpha)$, serving as probability amplitude of the field, with r = distance from origin, e = elementary charge, ε = electric constant and ρ = model specific constant. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:

1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma(1/3)|\Gamma(-1/3)| \approx \alpha^{-1}/(4\pi)$,

2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e = 1.509\left(y_l^m\right)^{-1/3}\Pi_{k=0}^{n}\alpha\text{^}(-1/3^k) \qquad (1)$$

with n = {0;1;2;..}, W_e = energy of electron, W_n= energy of particle n and y_l^m being a function of the spherical harmonics. For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ. The factor 1.509 is related to angular momentum |J| = 1/2.

The particles are interpreted as some kind of standing electromagnetic wave originating from a rotating electromagnetic field with the E-vector pointing towards the origin. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity. The terms for calculating energy do not distinguish between charged and neutral particles and have to be considered a first approximation, accurate only within order of magnitude of the spread of energies of particle families. Typical relative error of calculated parameters compared to experimental values is in a range of ± 0.01, within the same range the approximations made below are valid.

Interpreting the direct interaction between wave functions ("overlap") of two particles as strong interaction provides a method to distinguish leptons from hadrons.

Many details of the model still have to be worked out, yet the basic equations presented below are considered sufficient proof for relation (1) to be more than mere coincidence.

2 Results

2.1 Calculation of energy - point charge

To calculate particle energies the integral over the electrical field E of a point charge is used as a first approximation. However, it can not be expected that the expression derived from Coulomb's law for two interacting particles can be used unaltered and it will be demonstrated below that a factor 4π is needed as modification to yield a half integral angular momentum, giving:

$$W_{Coul,n} = \quad 4\pi \int_0^\infty \varepsilon_0 E(r)^2 d^3 r \quad = \quad 4\pi \int_0^\infty \frac{e^2}{4\pi\varepsilon_0 r^2} dr \quad = \quad 4\pi b_0 \int_0^\infty r^{-2} dr \qquad (2)$$

with $b_o = e^2/(4\pi\varepsilon_0)$ used for brevity.
The field E is modified with a function

$$\Psi(r) = \exp(-\{(\sigma\, \tau\, b_0^2\, r^{-3}) + [(\sigma\, \tau\, b_0^2\, r^{-3})^2 - 4\, \tau\, b_0^2\, r^{-3}]^{0,5}\} /2) \qquad (3)$$

The first term, $\exp(-\sigma\, \tau\, b_0^2\, r^{-3})$, avoids divergence of the E-field for $r \rightarrow 0$, the part in square brackets provides an integration limit, r_l, where the root term equals zero. r_l of particle n can be given by:

$$r_{l,n} = (\sigma^2 \tau_n\, b_0^2/4)^{1/3} \qquad (4)$$

providing a boundary condition for the problem.
Coefficient σ is a constant (σ = 1.76E+8[-]) related to constant angular momentum J (see below), τ is a parameter representing particle energy, $\tau_n \sim W_n^{-3}$. The coefficient τ_{n+1} of a particle can always be expressed by a term multiplying the coefficient of its predecessor n (defined in this work by $W_n < W_{n+1}$) with a parameter α_{n+1}: $\tau_{n+1} = \tau_n\, \alpha_{n+1}$. In general for the coefficient of particle n a partial product is formed relative to a reference particle, chosen here to be the electron, τ_e (electron coefficient τ_e = 1.68E+6 [m/J^2]):

$$\tau_n = \quad \tau_e \Pi_{k=0}^{n} \alpha_k \; = \tau_e\, \Pi_n \qquad (5)$$

In all integrals over Ψ(r) given below equ. (6) may be used as approximation for (3) up to $r = r_l$ with relative error << 0.01:

$$\Psi_n(r < r_l) \approx \exp(-\sigma\, \tau_n\, b_0^2\, r^{-3}) \; = \exp(-\beta_n/2\; r^{-3}) \qquad (6)$$

where $\beta_n = 2\, \sigma\, \tau_n\, b_0^2$ is used for brevity. The factor 2 takes into account, that Ψ(r) appears squared in the integrals below.
There are four closely related integrals over the approximation of Ψ(r) according to equ. (6) that are of interest to the problem:

$$\int^{r_l} \Psi(r)^2 r^{-(m+1)} dr \quad = \Gamma(m/3,\; \beta/r_L^3)\;\; \beta^{-m/3}/3 \qquad (7)$$

with m = {-1;0;1;2;}. The term $\Gamma(m/3, \beta/r_l^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind with s = m/3 and $x = \beta/r_l^3$ as lower integration limit:

$$\Gamma(s,x) = \quad \int_x^\infty t^{s-1} e^{-t}\, dt \qquad (8)$$

It follows from the boundary condition (4) that the integration limit $x = \beta/r_l^3$ has to be a constant for all particles:

$$\beta_n/r_{l,n}^3 = 2\sigma\tau_n b_0^2/\, r_{l,n}^3 = 8/\sigma \qquad (9)$$

For m = {1;2} $\Gamma(m/3, \beta/r_l^3)) \rightarrow \Gamma(m/3)$ gives a sufficient approximation for the equations of interest here and will be used below. For m = {-1;0} the integrals (7), (8) depend critically on the integration limit and have to be integrated numerically.
The integral for m = 1 is needed to calculate $W_{Coul,n}$. Inserting (6) and (7) in equ. (2) will turn out:

$$W_{Coul,n} = 4\pi \int_0^\infty \varepsilon_0 E(r)^2 \Psi_n(r)^2 d^3r = 4\pi b_0 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr = 4\pi\, b_0\, \Gamma(1/3)\, \beta_n^{-1/3}/3 \qquad (10)$$

Equation (10) is the source of $\tau_n \sim W_n^{-3}$. Through equ. (4) the relations $\tau_n \sim r_{l,n}^3$ and $W_n \sim r_{l,n}^{-1}$ hold.

The factor 4π added in equ. (2) may be derived by applying a semi-classical approach for angular momentum J, using J = r x p(r) = r W_n(r) /c_0 (assuming $W_{kin,n}$ = 1/2 W_n):

$$|J| = \int_0^{r_{l,n}} J_n(r)dr = 4\pi \frac{b_0}{c_0} \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-1} dr \qquad (11)$$

From (7), (8) follows for m = 0:

$$\int_0^{r_{l,n}} \Psi(r)^2 r^{-1} dr = 1/3 \int_{8/\sigma}^\infty t^{-1} e^{-t} dt = 5.447 \approx \alpha^{-1}/8\pi \qquad (12)$$

yielding the constant $\alpha^{-1}/8\pi$ for all particles. Inserting (12) in (11) provides a half integer angular momentum, |J| = 1/2:

$$|J| = 4\pi \frac{b_0}{c_0} \frac{\alpha^{-1}}{8\pi} = 1/2\ [\hbar] \qquad (13)$$

Analogous to the postulate for neutral particles to be composed of volume elements of opposite charge, integer spin particles as well as particles with J = 3/2 are supposed to be composed of a combination of half integer contributions of angular momentum J = ± 1/2, adding up accordingly.

2.2 Calculation of energy - photon

For m = -1 equations (7), (8) give a relation between radii and Euler-integral:

$$r_{x,n} = \int_0^{r_{x,n}} \Psi_n(r)^2 dr = \beta_n^{1/3}/3 \int_{\beta/r_{x,n}^3}^\infty t^{-4/3} e^{-t} dt \qquad (14)$$

Using the value of the Compton wavelength, λ_C, in the term for the energy of a photon gives hc_0/λ_C. With equ. (14) λ_C can be given by:

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr = \beta_n^{1/3}/3 \int_{\beta/\lambda_{C,n}^3}^\infty t^{-4/3} e^{-t} dt \approx \beta_n^{1/3}/3\ \ 18\pi \left| \Gamma(-1/3) \right| \qquad (15)$$

According to (10) particle energy is proportional to $\beta_n^{-1/3}$ and $\lambda_{C,n} \sim \beta_n^{1/3}$ has to hold, requiring the lower integration limit of the Euler integral and the factor ≈ 18π to be a constant for all particles. Energy of a photon can be expressed by:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} = \frac{3hc_0}{18\pi |\Gamma(-1/3)| \beta_n^{1/3}} \qquad (16)$$

2.3 Relation of integrals for $W_{Coul,n}$ and $W_{Phot,n}$ with α

The energy of a particle has to be the same in both photon and point charge description. From (10) and (16) follows:

$$W_{Coul,n} = W_{Phot,n} = 4\pi b_0\, \Gamma(1/3)\, \beta_n^{-1/3}/3 = \frac{3\,hc_0}{18\,\pi\,|\Gamma(-1/3)|\,\beta_n^{1/3}} \tag{17}$$

which my be expanded by 2π to transform h into ħ and rearranged to emphasize the relationship of the gamma functions ($\Gamma(1/3) = 2.679$; $|\Gamma(-1/3)| = 4.062$) with α, giving:

$$4\pi\,\Gamma(1/3)\,|\Gamma(-1/3)| \approx \frac{2\pi\,9\,hc_0}{2\,\pi\,18\pi\,b_0} = \frac{\hbar\,c_0}{b_0} = \alpha^{-1} \tag{18}$$

2.4 Coefficient 1.509 and related parameters

It is unclear if equation (18) can be used to directly link α with the quantization condition given in (1). However, the first term in (1), $W_\mu/W_e = 206.8 = 1.509\ \alpha^{-1}$ is within the accuracy of the calculations identically to the factor determing the integration limit, $1.501\ \alpha^{-1} \approx 1.5\ \alpha^{-1}$.
According to equation (14) $r_{l,n}$ may be given by :

$$r_{l,n} = \int_0^{r_{l,n}} \Psi(r)^2\,dr = \beta_n^{1/3}/3 \int_{8/\sigma}^{\infty} t^{-4/3} e^{-t}\,dt \approx 1.501\ \alpha^{-1}\,|\Gamma(-1/3)|\ \beta_n^{1/3}/3 \tag{19}$$

Consequently the equivalent term from (1) will cancel in the expression for $r_{l,\mu}$ (note: $W_n \sim 1/r_{l,n}$) :

$$r_{l,e} \approx 1.5\ \alpha^{-1}\,|\Gamma(-1/3)|\,\beta_e^{1/3}/3 \tag{20}$$

$$r_{l,\mu} \approx 1.5^{-1}\ \alpha^{+1}\,[1.5\ \alpha^{-1}\,|\Gamma(-1/3)|\,\beta_e^{1/3}/3\,] = |\Gamma(-1/3)|\,\beta_e^{1/3}/3 = 1.5\ \alpha^{-1}\,|\Gamma(-1/3)|\,\beta_\mu^{1/3}/3 \tag{21}$$

Assuming an identity of both terms, the value for $W_\mu/W_e = 1.509\ \alpha^{-1}$ will be used in all calculations as least biased value for $\approx 1.5\ \alpha^{-1}$, see discussion section. The coefficient σ is related to factor $1.509\ \alpha^{-1}$ by equ. (9) and (19) to be:

$$\sigma = 8\ r_{l,n}^3 / \beta_n = 8\ (1.509\ \alpha^{-1}\,|\Gamma(-1/3)|\,/3)^3 = 1.76\text{E+8}[-] = 68.3\ \alpha^{-3}\ [-] \tag{22}$$

Coefficients $1.5\ \alpha^{-1}$ and σ are part of the terms setting the integration limits in equ. (12), determining the value of J=1/2.
In analogy to σ the coefficient τ_e will be defined as

$$\tau_e = \rho\ 1.509^3\ \alpha^{-3} = \rho\ 3.4\ \alpha^{-3}\quad , \tag{23}$$

the coefficient ρ being the dimension bearing remainder. The actual value used in this work is obtained from a least square fit of energies of particles of the y_0^0 group, $\rho = 0.193\ [m/J^2]$.

2.5 Quantization with powers of 1/3ⁿ over α

To find a source for the quantization with powers of $1/3^n$ over α the ratio of the integrals used in (10) and (16) for the point charge and photon representation of energy may be examined.

$$Q(\psi_n) = \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2}\,dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2\,dr} = \frac{\Gamma(1/3)}{18\pi\ \ \Gamma(-1/3)\,\beta_n^{2/3}} \sim \frac{\Gamma(1/3)}{\Gamma(-1/3)}\,\frac{\alpha_0^{1/3}\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\alpha_1....\alpha_n} \tag{24}$$

with n = {0;1;2;..}. The term given by (24) is related to the boundary condition (4), (9) (see below) and via (10) and (16) to the square of particle energy $W_n^2 \sim \tau_n^{-2/3}$. The last expression of (24) is obtained by expanding the product $\Pi_n^{-2/3}$ included in $\beta_n^{-2/3}$ with $\Pi_n^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$

such as in equation (1) yields a distinct solution for $Q(\psi_n)$, $Q(\psi_n)$ being a function of coefficient α_n and α_0 only. By comparison with experimental data α_0 can be identified as $\alpha_0 = \alpha^3$ and $Q(\Psi_n)$ can in general be given by (n = {0;1;2;..}):

$$Q(\psi_n) \sim \frac{\Gamma(1/3)}{\Gamma(-1/3)} \frac{\alpha^1 \alpha^{1/3} \alpha^{1/9} .... \alpha \wedge (1/3^n)}{\alpha^3 \alpha^1 \alpha^{1/3} .... \alpha \wedge (3/3^n)} = \frac{\Gamma(1/3)}{\Gamma(-1/3)} \alpha \wedge (1/3^n)/\alpha^3 \tag{25}$$

where all intermediate particle coefficients cancel out. Equation (25) features α_{n+1} of equation (5) i.e. the reverse of the coefficient n in the partial product of (1) as well as the factor $\Gamma(-1/3)/\Gamma(1/3) = 1{,}51$. The coefficients for the product τ_n of (5) are given by ($0.29 = 1.51^{-3}$):

$$\tau_n = \tau_e\, 0.291 \quad \Pi_{k=0}^{n} \alpha \wedge (3/3^{\,k}) \quad = \quad \rho\, \alpha^{-3}\, \Pi_{k=0}^{n}\ \alpha \wedge (3/3^k) \quad = \ \rho\, \alpha^{-3}\, \Pi_n \qquad n = \{0;1;2;..\} \tag{26}$$

The term given by (24) is related to the boundary condition (9). Replacing $r_{l,n}$ in equation (9) by r, multiplying with $\Psi_n(r)^2$ and integrating, yields the following expression:

$$\beta_n \int_0^\infty \Psi_n(r)^2 r^{-3} dr \ = \ \frac{\Gamma(2/3)\beta_n}{3(\beta_n)^{2/3}} \ \sim \ \Pi_n \alpha_{n+1} \tag{27}$$

The integral $\int\Psi(r)^2 r^{-3}$ dr of (27) is directly proportional to $Q(\Psi_n)$, equation (24), via the term $\beta_n^{-2/3}$. Since $Q(\Psi_n) \sim \alpha_{n+1}$ equation (27) is proportional to $\Pi_n\ \alpha_{n+1} = \Pi_{n+1}$ and may be used to calculate particle coefficients τ_{n+1}.

2.6 Extension to non-spherical symmetry

Up to here only spherical symmetry is considered, introduced through equ. (2), (10). For a simple test if the model might be extendible to other symmetries equ. (24) is used. The integral over r^{-2} in $Q(\Psi_n)$ actually represents a volume integral, the factor 4π being included in equ. (2), (10) and thus implicitly in all related terms and coefficients. For non-spherically symmetric states an appropriate spherical harmonic factor, y_l^m, should be added to equ. (24)ff, given by the integral over non-normalized spherical harmonics i.e. the inverse of the square of the normalization factor N_l^m, corrected by the term 4π of Y_0^0 already contained in the equations:

$$y_l^m = \frac{1}{4\pi} \int P_l^m \cos(\vartheta) e^{im\varphi} P_l^m \cos(\vartheta) e^{-im\varphi} \sin(\vartheta)\, d\vartheta d\varphi = \frac{1}{4\pi (N_l^m)^2} \tag{28}$$

turning relation (25) into

$$Q(\Psi_n) \sim \ y_l^m \alpha \wedge (1/3^n)/\alpha^3 \tag{29}$$

and relation (26) into

$$\tau_n = y_l^m\ \tau_e\ 0.29 \quad \Pi_{k=0}^{n} \alpha \wedge (3/3^{\,k}) \quad = y_l^m \quad \rho\, \alpha^{-3}\, \Pi_{k=0}^{n}\ \alpha \wedge (3/3^k) \ = \ y_l^m\ \rho\ \alpha^{-3}\ \Pi_n \tag{30}$$

For the second spherical harmonic this gives $y_1^0 = 4\pi /(4\pi 3) = 1/3$, providing a second set of particle coefficients which is given by the coefficients according to (26) divided by 3. Table 1 shows results for y_0^0, y_1^0 relative to experimental values in col. 5. These are calculated according to equ. (10) using the coefficients of col. 4 in β_n. Relative energy values calculated by equ. (1) with the coefficients of col. 3 would be shifted by + 0.003 due to the electron becoming a reference particle.
For the transition from y_0^0 to y_1^0 the factor 1/3 in the coefficients τ (col. 4) appears as $3^{-1/3}$ =1.44 in the coefficients for energy ratio (col. 3). A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.
Included is a particle energy derived by expanding the model to energies below the electron with a coefficient of α^3 in equ. (1): $W_\nu / W_e = 1.509\ \alpha^3$. This gives a state with energy 0.3eV (for y_0^0) which is in a range expected for a neutrino [6].

	$W_{n,Lit}$ [MeV]	$\Pi_{k=0}^{n}$ α^(-1/3^k) equ (1)	$\tau/\rho = \alpha^{-3} * \Pi_{\tau,n}$ equ (30)	W_{calc}/W_{Lit} equ(10)	J	r_l [10E-15m]
ν	3E-7 *	$\boldsymbol{\alpha^{+3}}$	$\boldsymbol{\alpha^{-3}\alpha^{-9}}$	**-**	**1/2**	**1.5E+10**
e+-	**0.51**	Reference	$\boldsymbol{\alpha^{-3}}$	**0.997**	**1/2**	**8877**
μ+-	**105.66**	$\boldsymbol{\alpha^{-1}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}}$	**0.997**	**1/2**	**42.9**
π+-	*139.57*	*1.44* α^{-1}	$\alpha^{-3}\alpha^{3}/3$	*1.089*	*0*	*29.8*
K	495				0	
η 0	**547.86**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}\alpha^{1}}$	**0.992**	**0**	**8.3**
ρ0	*775.26*	*1.44* $(\alpha^{-1}\alpha^{-1/3})$	$(\alpha^{-3}\alpha^{3}\alpha^{1})/3$	*1.011*	*1*	*5.8*
ω0	*782.65*	*1.44* $(\alpha^{-1}\alpha^{-1/3})$	$(\alpha^{-3}\alpha^{3}\alpha^{1})/3$	*1.001*	*1*	*5.8*
K*	894				1	
p+-	**938.27**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3}}$	**1.000**	**1/2**	**4.8**
n	**939.57**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3}}$	**0.999**	**1/2**	**4.8**
η'0	958				0	
Φ'0	1019				1	
Λ0	**1115.68**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3}\alpha^{1/9}}$	**1.009**	**1/2**	**4.0**
Σ0	**1192.62**	$\boldsymbol{\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}}$	$\boldsymbol{\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}}$	**1.003**	**1/2**	**3.8**
Δ	**1232.00**	$\boldsymbol{\alpha^{-3/2}}$	$\boldsymbol{\alpha^{-3}\alpha^{9/2}}$	**1.001**	**3/2**	**3.7**
Ξ	1318				1/2	
*Σ*0*	*1382.80*	*1.44* $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$	$(\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3})/3$	*0.978*	*3/2*	*3.3*
Ω-	*1672.45*	*1.44* $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$	$(\alpha^{-3}\alpha^{3}\alpha^{1}\alpha^{1/3}\alpha^{1/9})/3$	*0.971*	*3/2*	*2.8*
tau+-	*1776.82*	*1.44* $(\alpha^{-3/2})$	$(\alpha^{-3}\alpha^{9/2})/3$	*1.001*	*1/2*	*2.5*

Table 1: Particles up to tau energy; calculated values for $\mathbf{y_0^0}$ **(bold)**, *y_1^0 (italic)* ; col. 2: energy values from literature [5] except *: calculated from model; Exponent of -3/2, 9/2 for Δ and tau is equal to the limit of the partial products in (1) and (30); r_l calculated according to (4);

3 Discussion

3.1 Relation to standard model and classical quantum mechanics

The standard model classifies particles into leptons, considered to be the fundamental "elementary particles" and hadrons, composed of two (mesons) or three (baryons) quarks. In the model presented the y_0^0 and y_1^0 groups each include all three particle types. Mesons constitute a distinct group of particles due to their integer angular momentum which is considered to be a combination of half-integer contributions in both models.

In the standard model leptons are characterized by not being subject to strong interaction. Neutrinos, electron and muon are the particles of lowest mass which in itself might provide an explanation for this quality. The tauon however is outstanding by possessing a mass almost twice that of the proton and by major decay channels involving hadrons.

In this model, on the length scale of particle radius, the wave functions of two particles should start to overlap and exert some kind of direct interaction. As demonstrated in table 1, last column, for hadrons the model yields particle radius in the range of femtometer, the characteristic scale for strong interaction and it seems likely to identify strong interaction with the interaction of wave functions. Interaction via overlapping of wave functions constitutes the basis of chemistry and has been examined extensively [7]. In general wave functions are signed (not to be confused with electrical charge), for particles above the ground state regions

of different sign exist, separated by nodes. There are two major requirements for effective interaction:
1) Comparable size and energy of wave functions,
2) sufficient net overlap: In the overlap region of two interacting wave functions sign should be the same (bonding) or opposite (antibonding) in all overlapping regions. If regions with same and opposite sign balance to give zero net overlap, no interaction results.
From condition 1) and Table 1 it is obvious that the wave functions of neutrino and electron/positron will not show effective interaction with hadrons. In the case of the tauon the second rule is crucial. According to this model the tauon is at the end of the partial product series for y_1^0 and should consequently exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign. Though having particle size and energy in the same order of magnitude as other hadrons, such as the proton, the frequent change of sign of the tauon wave function will prohibit net overlap and effective interaction.
It is not obvious how to treat e.g. the muon, pion or delta particle within this picture and a more detailed solution for the wave function is needed for this. In general a strict classification into “leptons” and “hadrons” should not be expected according to this model and experimental evidence might at least partly be biased by the limitation of available scattering partners.

The relation of this model to classical quantum mechanics may be given by interpreting Ψ(r) as probability amplitude applied to a field instead of a particle. This implies that concepts such as orthonormalization and calculation of eigenvalues may not be applicable on the level of the differential equation. Properties have to be calculated by integration over the spatial extent of the field.
The quantization condition itself is not exclusive. The special solution of (25) coincides with the rest mass of particles of sufficiently high mean lifetime to be experimentally observable but does not prohibit the existence of particles with any other mass.
As for the number of parameters needed to calculate energy states the model resembles the simplicity of basic quantum mechanical models, relying essentially on $4\pi\, b_0 = e^2 /\varepsilon$ and $J = 1/2$ to yield the expression (1). The second parameter ρ is needed to transform the relative energy scale of (1) into an absolute one.

3.2 Accuracy

The values calculated for y_0^0 agree within ± 0.01 with experimental data. There are two major causes preventing a significant improvement of accuracy.
1) Especially in the case of particle families effects on top of the relations given in this work have to play a role to explain different energy levels for differently charged particles. This limits accuracy and the possibility to precisely identify candidates for calculated energies (e.g. both ρ^0 and ω^0 are given for $1.44\ \alpha^{-1}\alpha^{-1/3}$ in tab. 1).
If possible, particles chosen for y_0^0 in table 1 are of charge ± 1. In cases such as Σ with three energy levels, the intermediate energy level is chosen as reference. For y_1^0 particles of the same charge as their y_0^0 equivalent are preferred in table 1.
Remaining particles in table 1 may be explained by higher excitation or linear combinations of lower states. Expressions for linear combinations are expected to be complex due to the differences to conventional quantum mechanics addressed in 3.1. At the present level of understanding and accuracy of the model it is considered too speculative to attempt to assign additional particle states.
Conversely, energy states belonging to higher terms of the y_0^0, y_l^0 partial products may be missing an identifyable experimental counterpart. The next y_0^0 particle following Σ^0 is expected at 1217 MeV, the next y_l^0 particle following Ω^- is expected at 1726 MeV with J = 3/2. At least for the latter there exists a resonance at 1720 MeV with J = 3/2 [8] as possible candidate.
2) The second effect is due to ambiguity in fitting model parameters to experimental values. The results presented in this article are calculated using $1.509\ \alpha^{-1}$ as value for $\approx 1.5\ \alpha^{-1}$ originating from direct experimental data of the energy ratio of μ and e and close to the ratio of the Γ-coefficients of equation (24). This value is used to calculate σ via equ. (22). Parameter ρ is calculated using a least square fit of energies of y_0^0 particles using equ. (10). Replacing the approximation (7) with the exact term (3) in equation (10) or

choosing other sets of fitting particles may change results by roughly ± 0.01.

References

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LibreCalc files:

1) Numerical calculation of particle energies:
Num Calc W.ods
http://doi.org/10.5281/zenodo.570158
2) Results of tables:
Results.ods
http://doi.org/10.5281/zenodo.570159
3) Numerical calculation of Euler integrals:
Euler.ods
http://doi.org/10.5281/zenodo.570160